SOLID STATE PHYSICS I

EXAM February 06, 2002

◇ Do not forget to write your full name and student number on each sheet.
◇ Please use separate sheets for each of the problems.
◇ The answers may be given in dutch

Problem 1

Consider a one dimensional material with one monovalent atom A per primitive cell. The lattice constant is a.

a) What is the density of states per unit length in k-space.
b) Derive an expression for the Fermi energy within the free electron approximation.
c) Draw the energy versus wave vector diagram, and indicate the Fermi energy and the first Brillouin zone boundaries.

Let $\Psi(x) = \sum_k C_k exp(ikx)$ be the wave function of an electron in the crystal. The central equation relates the coefficients C_k to the Fourier components U_G of the potential energy: $(\lambda_k - \epsilon)C_k + \sum_G U_G C_{k-G} = 0$, with λ_k the free electron energy $\hbar^2 k^2/2m$. Let the potential energy be $U(x) = 2U cos(2\pi x/a)$.

d) Show that the Fourier components U_G of this potential are equal to U.
e) Use the central equation to calculate the energy gap for k=G/2.
f) i. Is this material a metal or an insulator (explain !) ?
Would the answer to i. change when
ii. The atom A is divalent (explain !) ?
iii. There is a strong on-site coulomb interaction between the electrons (explain !) ?

Problem 2

a) The magnetic susceptibility of a paramagnetic material is given by the Curie law: $\chi = \frac{M}{B} = \frac{C}{T}$. Consider a ferromagnet in the mean field approximation, where, in addition to the applied field B_a, the surrounding paramagnetic atoms cause an exchange field $B_E = \lambda M$ acting on every moment.
Use Curie's law to derive an expression for the susceptibility in the mean field approximation.
b) Sketch the magnetization of a ferromagnet as a function of temperature.
c) Consider Iron, for which the mean field parameter $\lambda = 2045$, and the Curie constant $C = 0.51$ K. At which temperature does Iron order ferromagnetically ?

d) What are magnons ? Sketch the dispersion relation.

e) The low energy part of the dispersion relation may be approximated by $\hbar\omega = (2JSa^2)k^2$. Derive an expression for the density of magnon modes per unit energy range, $D(\omega)$, using this approximation.

f) Show that at low temperature the saturation magnetization $M(T)$ varies with temperature as $M(0)(1 - C \cdot T^{3/2})$, where C is a constant.

note: $\int_0^\infty \frac{\sqrt{x}}{e^x - 1} = 2.317$; plank's distribution: $\langle n(k) \rangle = (e^{\hbar\omega_k/k_bT} - 1)^{-1}$

Problem 3

Consider a two dimensional square lattice with lattice constant a. The conventional cell, spanned by the vectors $\vec{a}_1 = (a, 0)$ and $\vec{a}_2 = (0, a)$, contains one type of atoms located at (1/4,1/4) and (3/4,3/4).

a) Calculate the angle between the 'planes' (10) and a (13), denoted in the system $\{\vec{a}_1, \vec{a}_2\}$.

b) Give a definition of the primitive cell.

c) Give the primitive vectors of a primitive cell for this lattice. How many atoms does the primitive cell contain ?.

d) What are the primitive lattice vectors of the corresponding reciprocal lattice ?

e) The dispersion relation for longitudinal phonons in this lattice is given by:

$$\omega_k^2 = \frac{4C}{m}\left(\sin^2 \frac{k_x a\sqrt{2}}{4} + \sin^2 \frac{k_y a\sqrt{2}}{4}\right)$$

What is the sound velocity for sound waves traveling in the $k_x = k_y$ direction ?

f) Sketch the thermal conductivity for a three dimensional insulating material as a function of temperature.
Which processes determine the thermal conductivity at high temperatures ?
What limits the thermal conductivity at low temperatures ?